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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

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Thursday 3 May 2012 (afternoon)

Examination code

2	2	1	2	–	7	3	0	3

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

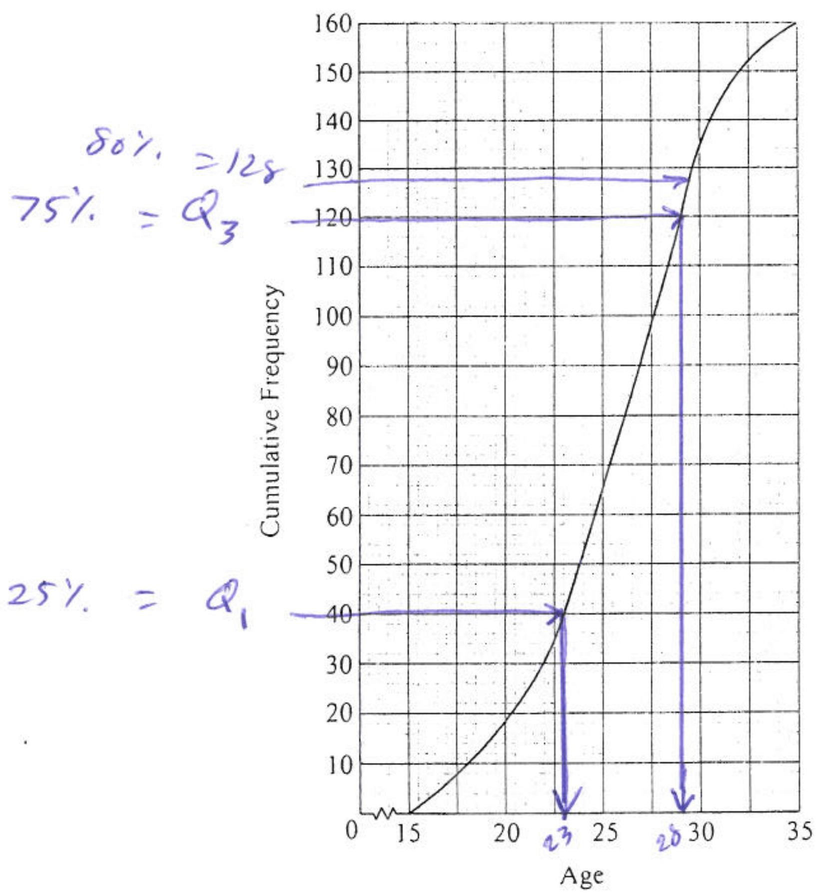
1. *[Maximum mark: 7]*

The ages of people attending a music concert are given in the table below.

Age	$15 \le x < 19$	$19 \le x < 23$	$23 \le x < 27$	$27 \le x < 31$	$31 \le x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	p	160

(a) Find p. $= 92 + 52 = 104$ *[2 marks]*

The cumulative frequency diagram is given below.



(This question continues on the following page)

(Question 1 continued)

(b) Use the diagram to estimate

(i) the 80th percentile;

(ii) the interquartile range. *[5 marks]*

i) $.8 \times 160 = 128.0$ This is cumulative frequency. Use it to find age.

$\boxed{\approx 29 \text{ years old}}$

ii) $IQR = Q_3 - Q_1$, upper quartile $= Q_3$, lower quartile $= Q_1$

$Q_3 = .75 \times 160 = 120 \Rightarrow 28$ years

$.5 \times 160 = 80$

$Q_1 = .25 \times 160 = 40 \Rightarrow 23$ years

$\boxed{IQR = 28 - 23 = 5 \text{ years of age}}$

2. *[Maximum mark: 6]*

Let A be a 2×2 matrix and B an $m\times n$ matrix, where $A=\begin{pmatrix}-2 & 0\\ 1 & 3\end{pmatrix}$ and $B=\begin{pmatrix}1 & 0 & 3\\ -1 & 3 & 0\end{pmatrix}$.

Row Column

(a) Write down the value of m and of n. *[2 marks]*

(b) Find AB. *[3 marks]*

(c) Let C be a $p\times4$ matrix. Given that the product BC exists, write down the value of p. *[1 mark]*

a). A , B

2×2 2×3

same

Value of m and of n = 2×3

b) $\begin{pmatrix}-2 & 0\\ 1 & 3\end{pmatrix}\begin{pmatrix}1 & 0 & 3\\ -1 & 3 & 0\end{pmatrix}=\begin{pmatrix}-2+0 & 0+0 & -6+0\\ 1-3 & 0+9 & 3+0\end{pmatrix}$

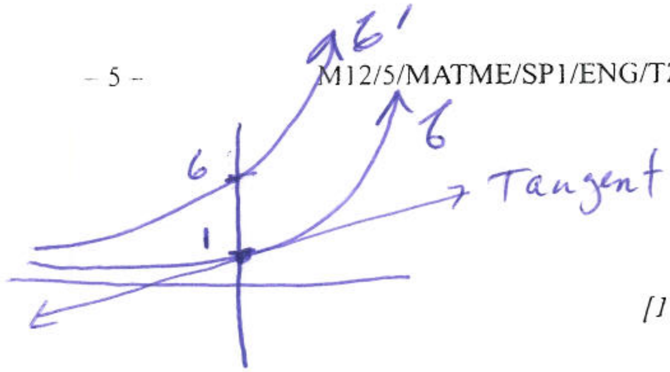
$AB=\begin{pmatrix}-2 & 0 & -6\\ -2 & 9 & 3\end{pmatrix}$

c) B C

2×3 $p\times4$

must be same if BC exists.

$\boxed{\therefore\ p=3}$



3. *[Maximum mark: 6]*

Let $f(x) = e^{6x}$.

(a) Write down $f'(x)$. *[1 mark]*

The tangent to the graph of f at the point $P(0, b)$ has gradient m.

(b) (i) Show that $m = 6$.

(ii) Find b. *[4 marks]*

(c) Hence, write down the equation of this tangent. *[1 mark]*

a) $f'(x) = 6e^{6x}$

b i) $f'(0) = 6e^{6\cdot 0} = \boxed{6 = m}$

b ii) $f(0) = e^{6\cdot 0} = \boxed{1 = b}$

c) Equation of tangent line: $y - y_1 = m(x - x_1)$

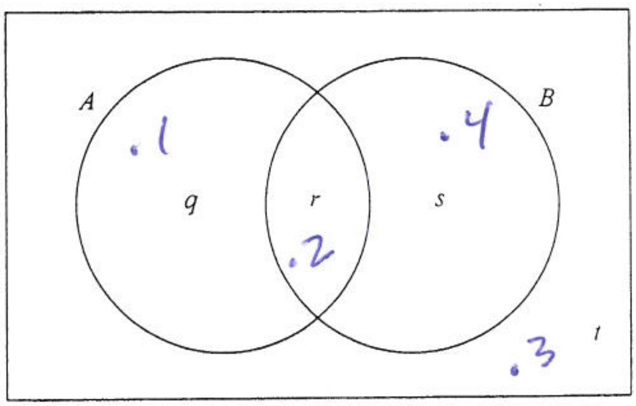
$\therefore\ y - 1 = 6(x - 0)$ @ $(0, 1)$

$y - 1 = 6x$

$\boxed{y_T = 6x + 1}$

4. *[Maximum mark: 7]*

Events A and B are such that $P(A) = 0.3$, $P(B) = 0.6$ and $P(A \cup B) = 0.7$.



The values q, r, s and t represent probabilities.

(a) Write down the value of t. $\boxed{t = 1 - .7 = .3}$ *[1 mark]*

(b) (i) Show that $r = 0.2$.

(ii) Write down the value of q and of s. $q = .3 - .2 = .1$, $s = .6 - .2 = .4$ *[3 marks]*

(c) (i) Write down $P(B') = 1 - P(B) = 1 - .6 = .4$

(ii) Find $P(A|B')$. IB Pocket *[3 marks]*

bi) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.7 = .3 + .6 - P(A \cap B)$

$P(A \cap B) = .9 - .7 = \boxed{0.2 = r}$

cii) $P(A|B') = \dfrac{P(A \cap B')}{P(B')}$

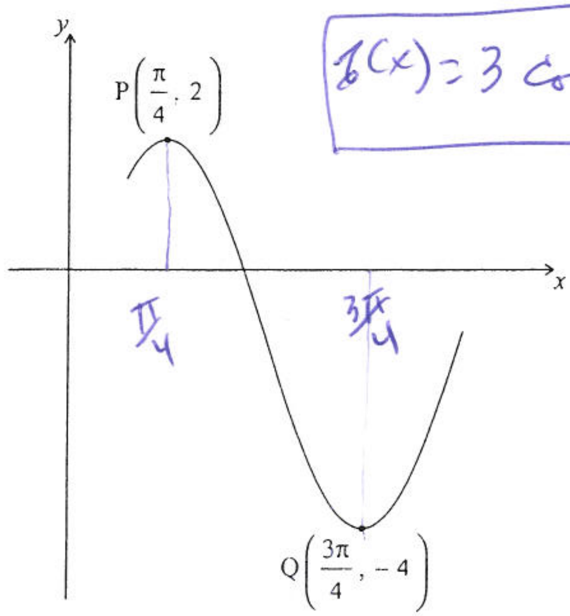
Note
$A = q, r$
$B' = q, t$
$A \cap B' = q$

$= \dfrac{.1}{.4} = \boxed{\dfrac{1}{4}}$

5. *[Maximum mark: 7]*

The diagram below shows part of the graph of $f(x) = a\cos(b(x-c)) - 1$, where $a > 0$.

$\boxed{f(x) = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right) - 1}$



The point $P\left(\frac{\pi}{4}, 2\right)$ is a maximum point and the point $Q\left(\frac{3\pi}{4}, -4\right)$ is a minimum point.

(a) Find the value of a. *[2 marks]*

(b) (i) Show that the period of f is π.

(ii) Hence, find the value of b. *[4 marks]*

(c) Given that $0 < c < \pi$, write down the value of c. *[1 mark]*

a) $a = \text{amplitude} = \left|\frac{\text{Max} - \text{Min}}{2}\right| = \left|\frac{2 - -4}{2}\right| = \boxed{3}$

bi) Period is 2 consecutive Max/Min or twice the size of Max to Min.
from $\frac{\pi}{4}$ to $\frac{3\pi}{4}$ is $\frac{\pi}{2}$. $\therefore \boxed{P = 2\,\frac{\pi}{2} = \pi}$

bii) $b = \frac{2\pi}{\text{Period}} = \frac{2\pi}{\pi} = \boxed{2}$

c) $\boxed{c = \frac{\pi}{4}}$ Note: $3\cos x$ starts @ Max @ the point $(0,3)$. The graph is shifted horizontally to the right $\frac{\pi}{4}$.

6. *[Maximum mark: 6]*

Given that $\int_0^5 \frac{2}{2x+5}\,dx = \ln k$, find the value of k.

$$\int_0^5 \frac{2}{2x+5}\,dx = 2\int_0^5 \frac{1}{2x+5}\,dx$$

$$= \cancel{2}\left.\frac{\ln(2x+5)}{\cancel{2}}\right]_0^5$$

$$= \ln(2\cdot 5+5) - \ln(2\cdot 0+5)$$

$$= \ln 15 - \ln 5$$

$$= \ln\frac{15}{5} = \boxed{\ln 3}$$

$$\ln 3 = \ln k$$

$$\boxed{\therefore k = 3}$$

Note: $\int \frac{1}{♡} = \frac{\ln ♡}{♡'}$

or use u-substitution

$$\boxed{\begin{gathered} u = 2x+5 \\ du = 2\,dx \end{gathered}}$$

 $= 1$

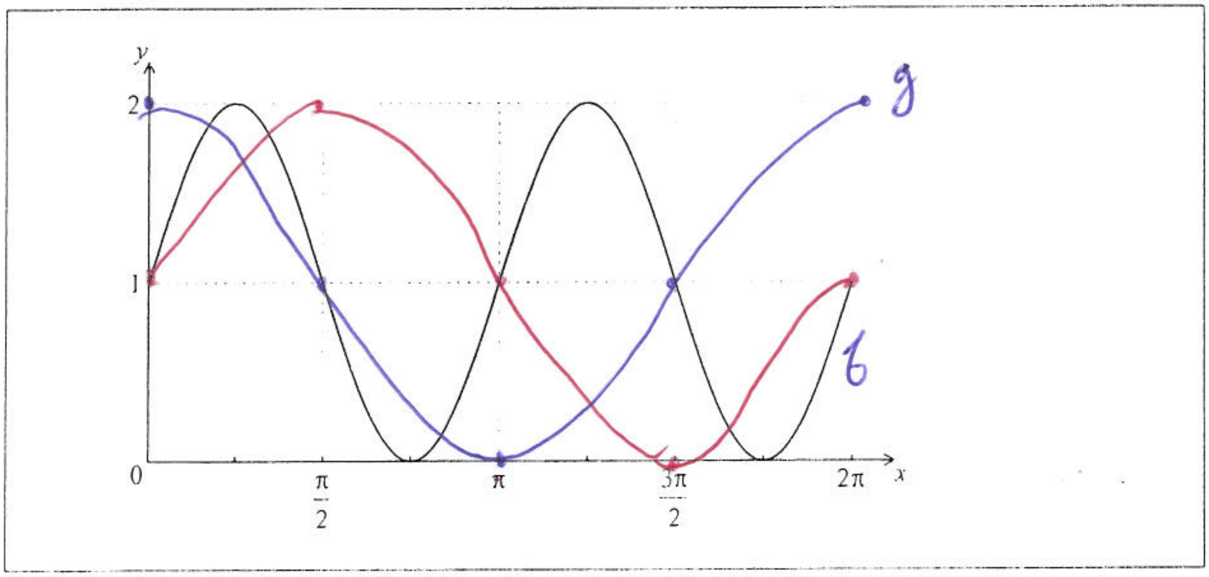
7. *[Maximum mark: 6]*

Let $f(x) = (\sin x + \cos x)^2$. $= \sin^2 x + 2\sin x\cos x + \cos^2 x = 1 + 2\sin x\cos x$

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$. $\boxed{1+\sin 2x}$ *[2 marks]*

IB packet.

The graph of f is shown below for $0 \le x \le 2\pi$.



(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$. *[2 marks]*

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$. ← x-direction ← y-direction

(c) Write down the value of p and a possible value of k. *[2 marks]*

c) $P = 2$

Note: $\sin 2x$ has period $= \pi$
$\sin x$ has period $= 2\pi$
$\therefore$ stretch by a factor of 2.
OR
You have $\sin 2x$ and you want $\sin x$
$\sin x = \sin \frac{2x}{2}$ ← This is a stretch horizontally by a factor of 2.

$K = -\frac{\pi}{2}$
moves the graph horizontally in negative x-direction.
Left $\frac{\pi}{2}$

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

A line L_1 passes though points $P(-1, 6, -1)$ and $Q(0, 4, 1)$.

(a) (i) Show that $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix} 0-{-1} \\ 4-6 \\ 1-{-1} \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

(ii) Hence, write down an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[3 marks]*

$r = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

↳ choose any point.

A second line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

(b) Find the cosine of the angle between $\overrightarrow{PQ}$ and L_2. *[7 marks]*

(c) The lines L_1 and L_2 intersect at the point R. Find the coordinates of R. *[7 marks]*

b) $\cos\theta = \dfrac{L_1 \cdot L_2}{|L_1||L_2|} = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}}{\sqrt{1^2+(-2)^2+2^2}\,\sqrt{3^2+0^2+(-4)^2}} = \dfrac{3-8}{\sqrt{9}\,\sqrt{25}} = \dfrac{-5}{15}$

$\boxed{\cos\theta = -\frac{1}{3}}$

c) L_1 L_2

$x = 0 + t = \boxed{1}$ $\quad x = 4 + 3s = \boxed{1}$

$y = 4 - 2t = \boxed{2}$ $\quad y = 2 + 0s = \boxed{2}$

$z = 1 + 2t = \boxed{3}$ $\quad z = -1 - 4s = \boxed{3}$

Same values so point of intersection $R = (1, 2, 3)$

Set $L_1 = L_2$ and solve for s, t.

$x \Rightarrow t = 4 + 3s \Rightarrow 1 = 4 + 3s \Rightarrow 3s = -3 \Rightarrow \boxed{s = -1}$

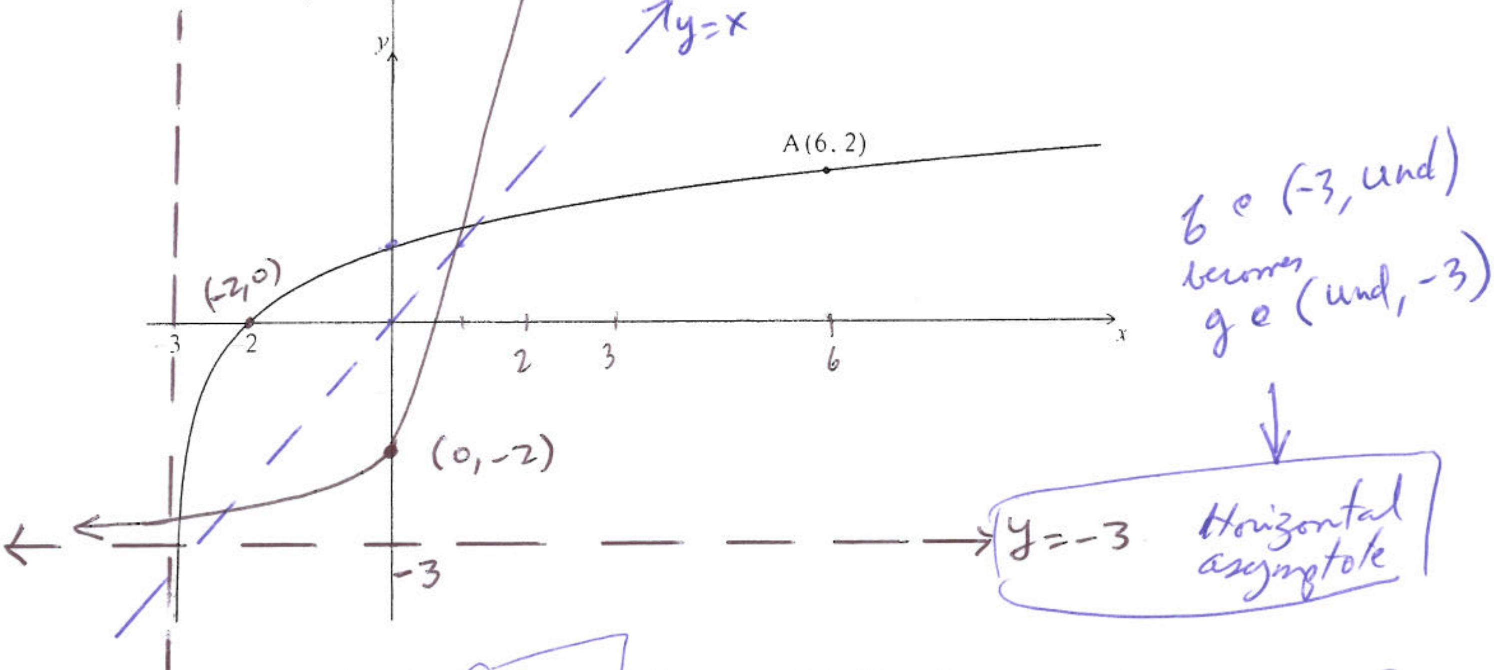
$y \Rightarrow 4 - 2t = 2 \Rightarrow 2t = 2 \Rightarrow \boxed{t = 1}$

$z \Rightarrow 1 + 2t = -1 - 4s$

*Do **NOT** write solutions on this page.*

[Maximum mark: 13]

Let $f(x) = \log_p(x+3)$ for $x > -3$. Part of the graph of f is shown below.



The graph passes through A(6, 2), has an x-intercept at $(-2, 0)$ and has an asymptote at $x = -3$.

$\log_p(x+3) = \frac{\log(x+3)}{\log p} \Rightarrow \frac{\log(6+3)}{\log p} = 2 \Rightarrow \frac{\log 9}{\log p} = 2 \Rightarrow \frac{\log 3^2}{\log p} = 2$

(a) Find p. *[4 marks]*

$\Rightarrow \frac{2\log 3}{\log p} = 2 \Rightarrow 2\log 3 = 2\log p \Rightarrow \log 3 = \log p \therefore \boxed{p = 3}$

The graph of f is reflected in the line $y = x$ to give the graph of g.

(b) (i) Write down the y-intercept of the graph of g. $(x, y) \rightarrow (y, x) \rightarrow$ reflection across $y = x$ line.

$(-2, 0) \rightarrow (0, -2)$

(ii) Sketch the graph of g, noting clearly any asymptotes and the image of A. *[5 marks]*

(c) Find $g(x)$. *[4 marks]*

$y = \log_3(x+3)$, Replace x for y and solve for y.

$3^{[x]} = 3^{[\log_3(y+3)]}$

$3^x = y + 3$

$\boxed{g(x) = y^{-1} = 3^x - 3}$

*Do **NOT** write solutions on this page.*

. *[Maximum mark: 15]*

In this question, you are given that $\cos\frac{\pi}{3} = \frac{1}{2}$, *and* $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

The displacement of an object from a fixed point, O is given by $s(t) = t - \sin 2t$ for $0 \le t \le \pi$.

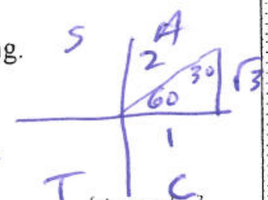
(a) Find $s'(t)$ $= 1 - 2\cos 2t$ *[3 marks]*

In this interval, there are only two values of t for which the object is not moving. One value is $t = \frac{\pi}{6}$.

$1 - 2\cos 2t = 0, \quad \cos 2t = \frac{1}{2}$

$2t = \frac{\pi}{3}$ and $\frac{5\pi}{3}$

Note: $\cos\theta$ is + in QI, IV

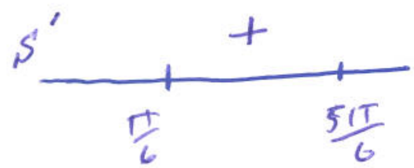


(b) Find the other value. $\boxed{t = \frac{\pi}{6} \text{ and } \frac{5\pi}{6}}$ *[4 marks]*

(c) Show that $s'(t) > 0$ between these two values of t. *[3 marks]*

(d) Find the distance travelled between these two values of t. *[5 marks]*

c.



$\frac{\pi}{2}$ is a value between $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.

$s'\left(\frac{\pi}{2}\right) = 1 - 2\cos\left(2 \cdot \frac{\pi}{2}\right) = 1 - 2\cos\pi = 1 - 2(-1) = 3$

$3 > 0$

d. distance $= \int |v(t)|\,dt = \int_{\pi/6}^{5\pi/6} 1 - 2\cos 2t \, dt$

$= t - \sin 2t \Big]_{\pi/6}^{5\pi/6}$

$= \left(\frac{5\pi}{6} - \sin 2 \cdot \frac{5\pi}{6}\right) - \left(\frac{\pi}{6} - \sin 2 \cdot \frac{\pi}{6}\right)$

$= \left(\frac{5\pi}{6} - \sin\frac{5\pi}{3}\right) - \left(\frac{\pi}{6} - \sin\frac{\pi}{3}\right)$

$= \left(\frac{5\pi}{6} + \frac{\sqrt{3}}{2}\right) - \left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right) = \frac{4\pi}{6} + \frac{2\sqrt{3}}{2}$

$= \frac{2\pi}{3} + \sqrt{3}$