Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics
# Higher level
# Paper 2

Walbdr Key

Wednesday 13 May 2015 (afternoon)

2 hours

Candidate session number

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

## Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks].**

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

## Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

**1.** [Maximum mark: 4]

The region $R$ is enclosed by the graph of $y = \mathrm{e}^{-x^2}$, the $x$-axis and the lines $x = -1$ and $x = 1$. Find the volume of the solid of revolution that is formed when $R$ is rotated through $2\pi$ about the $x$-axis.

$V = \int_{-1}^{1} \pi \left(e^{-x^2}\right)^2 dx$

GDC: $= 3.76$ (3.06)

**2.** [Maximum mark: 4]

The finishing times in a marathon race follow a normal distribution with mean 210 minutes and standard deviation 22 minutes.

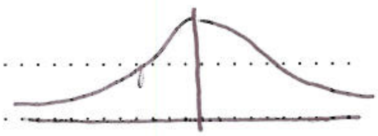
(a) Find the probability that a runner finishes the race in under three hours. [2]

The fastest 90 % of the finishers receive a certificate.

(b) Find the time, below which a competitor has to complete the race, in order to gain a certificate. [2]

$\mu = 210, \quad \sigma = 22$

a) 3 hours = 180 min.

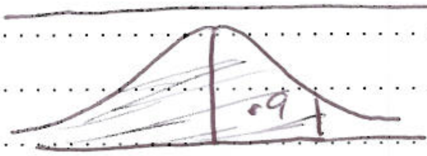



$X \sim N(210, 22^2)$

$P(X < 180) = 0.0863 \rightarrow 2^{nd}$ vars, normalcdf

Lower: −100
upper: 180
$\mu$ : 210
$\sigma$ : 22

b)




$2^{nd}$ vars, invnorm
area: 0.9
$\mu$ : 210
$\sigma$ : 22

$P(X < T) = 0.9$

$T = 238.2\ldots$

$\boxed{T = 238 \text{ mins}}$

**3.** [Maximum mark: 5]

A mosaic is going to be created by randomly selecting $1000$ small tiles, each of which is either black or white. The probability that a tile is white is $0.1$. Let the random variable $W$ be the number of white tiles.

(a) State the distribution of $W$, including the values of any parameters. [2]

(b) Write down the mean of $W$. [1]

(c) Find $\mathrm{P}(W > 89)$. [2]

a) This is a binomial distribution w/ parameters:
1) $n = 1000$, fixed # of trials
2) $p = 0.1$, probability of success

$W \sim B(n, p) \Rightarrow W \sim B(1000, 0.1)$

b) $\mu = np \Rightarrow \mu = 1000(0.1) = \boxed{100}$

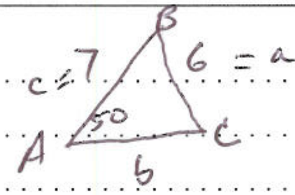
c) $P(W > 89) = 1 - P(W \leq 89$
$= 1 - \text{binomcdf}(100, 0.1, 89)$
$\boxed{= 0.867}$ (3 s.f.)

2nd var, binomcdf
trials: 1000
p: 0.1
x: 89

**4.** [Maximum mark: 6]

A triangle ABC has $\hat{A} = 50°$, $\mathrm{AB} = 7\,\mathrm{cm}$ and $\mathrm{BC} = 6\,\mathrm{cm}$. Find the area of the triangle given that it is smaller than $10\,\mathrm{cm}^2$.

Note: Check to see if this is the ambiguous case. If so redraw triangle to show 2 Δ's.




Method 1:

$c^2 = a^2 + b^2 - 2ab\cos C$ Law of Cos in packet

$\therefore a^2 = c^2 + b^2 - 2cb\cos A$

$6^2 = 7^2 + b^2 - 2(7)b\cos 50$

$b^2 - 14b\cos 50 - 13 = 0 \rightarrow$ Think $ax^2 + bx + c = 0$ use quadratic formula.

$$b = \frac{+14\cos 50 \pm \sqrt{(-14\cos 50)^2 - 4(1)(-13)}}{2}$$

GDC:

$b_1 = 7.191282137 \rightarrow$ store D

$b_2 = 1.807744398 \rightarrow$ store E

Area of $\Delta = \frac{1}{2} bc\sin A \rightarrow$ formula packet.

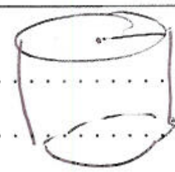
$A = \frac{1}{2}(D)(7)\sin 50 = 19.3 > 10 \therefore$ reject.

$A = \frac{1}{2}(E)(7)\sin 50 = \boxed{4.85} < 10 \therefore$ accept.

**5.** [Maximum mark: 5]

A bicycle inner tube can be considered as a joined up cylinder of fixed length $200\,\text{cm}$ and radius $r\,\text{cm}$. The radius $r$ increases as the inner tube is pumped up. Air is being pumped into the inner tube so that the volume of air in the tube increases at a constant rate of $30\,\text{cm}^3\,\text{s}^{-1}$. Find the rate at which the radius of the inner tube is increasing when $r = 2\,\text{cm}$.

Given: $\frac{dv}{dt} = 30$ , $h = 200$




Find: $\frac{dr}{dt}$ when $r = 2$

$V = \pi r^2 h$ Volume of cylinder in formula packet.

$\frac{dv}{dt} = 2\pi r \frac{dr}{dt} h$ Note: $h$ is constant

$30 = 2\pi(2)\frac{dr}{dt}(200)$

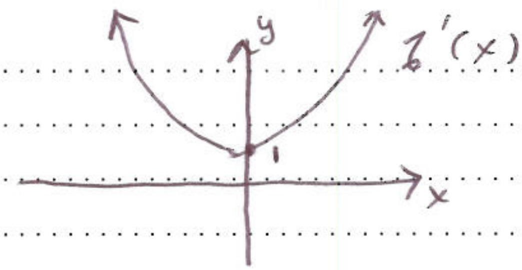
$\boxed{\frac{dr}{dt} = \frac{3}{80\pi} = 0.0119 \text{ cm/sec.}}$

**6.** [Maximum mark: 4]

A function $f$ is defined by $f(x) = x^3 + e^x + 1$, $x \in \mathbb{R}$. By considering $f'(x)$ determine whether $f$ is a one-to-one or a many-to-one function.

$f'(x) = 3x^2 + e^x$

consider the graph of $f'(x)$ using G.D.C.




since $f'(x) > 0$, that is, positive for all values of $x$,

then $f(x)$ is a function that is always increasing. Thus one-to-one function.

Note: If $f'(x)$ changes direction from positive to negative or vice vasa then it is considered a many-to-one function.

**7.** [Maximum mark: 7]

The random variable $X$ follows a Poisson distribution with mean $m \neq 0$.

(a) Given that $2\mathrm{P}(X = 4) = \mathrm{P}(X = 5)$, show that $m = 10$. [3]

(b) Given that $X \leq 11$, find the probability that $X = 6$. [4]

$\frac{m^x e^{-m}}{x!}$ → formula in packet.

$2P(X=4) = P(X=5)$

$\frac{2\, m^4 e^{-m}}{4!} = \frac{m^5 e^{-m}}{5!}$ Solve for m.

$\frac{2}{4!} = \frac{m}{5!} \Longrightarrow \boxed{m=10}$

b) $P(X=6 \mid X \leq 11)$

$P(X=6 \mid X \leq 11) = \frac{P(X=6 \cap X \leq 11)}{P(X \leq 11)}$

$= \frac{P(X=6)}{P(X \leq 11)}$

$= \frac{\text{poisson pdf}(10, 6)}{\text{poisson cdf}(10, 11)}$

$\boxed{= 0.0905}$

**8.** [Maximum mark: 8]

Let $v = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ and $w = \begin{pmatrix} 4 \\ \lambda \\ 10 \end{pmatrix}$.

(a) Find the value of $\lambda$ for $v$ and $w$ to be parallel. → same direction [2]

(b) Find the value of $\lambda$ for $v$ and $w$ to be perpendicular. → $v \cdot w = 0$ [2]

(c) Find the two values of $\lambda$ if the angle between $v$ and $w$ is $10^\circ$. [4]

a) let $\begin{pmatrix} 4 \\ \lambda \\ 10 \end{pmatrix} = t\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \Rightarrow 4 = 2t \Rightarrow t = 2$

$\lambda = 3t \Rightarrow \lambda = 3(2) = 6$

$\boxed{\therefore \lambda = 6}$

b) $\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ \lambda \\ 10 \end{pmatrix} = 8 + 3\lambda + 50 = 0 \Rightarrow \boxed{\lambda = \frac{-58}{3}}$

c) $\cos\theta = \frac{v \cdot w}{|v||w|}$ → formula in packet.

$$\cos 10 = \frac{3\lambda + 58}{\sqrt{2^2+3^2+5^2}\sqrt{4^2+\lambda^2+10^2}}$$

$$\cos 10 = \frac{3\lambda + 58}{\sqrt{38}\sqrt{116+\lambda^2}}$$

$3\lambda + 58 = \sqrt{38}\sqrt{116+\lambda^2}\cos 10$

GDC: Find intersections → let $y_1 = 3\lambda + 58$

$y_2 = \sqrt{38}\sqrt{116+\lambda^2}\cos 10$

$\boxed{\lambda = 3.73,\ 8.76}$

**9.** [Maximum mark: 7]

Find the equation of the normal to the curve $y = \dfrac{e^x \cos x \ln(x+e)}{(x^{17}+1)^5}$ at the point where $x = 0$.

In your answer give the value of the gradient, of the normal, to three decimal places.

Consider $y = uvw$ use product rule

$y' = u'vw + uv'w + uvw'$

$$y' = \frac{\left[e^x \cos x \ln(x+e) + e^x(-\sin x)\ln(x+e) + e^x \cos x\left(\frac{1}{x+e}\right)\right]\left[(x^{17}+1)^5\right] - \left[\left(e^x \cos x \ln(x+e)\right)\left(5(x^{17}+1)^4(17x^{16})\right)\right]}{\left((x^{17}+1)^5\right)^2}$$

Think: Quotient Rule.

$$y'(0) = \frac{\left(1+0+\frac{1}{e}\right)(1) - (1 \cdot 0)}{1}$$

$\boxed{y'(0) = 1 + \frac{1}{e} = \frac{e+1}{e} = 1.37 \text{ value of gradient}}$

$\boxed{\therefore \text{gradient of normal} = -\frac{e}{e+1} = -0.731}$

Eq. of normal: $y - y_1 = m(x - x_1)$

$y(0) = \ln e = 1$

$\therefore y - 1 = -\frac{e}{e+1}(x - 0)$ or $y - 1 = -0.731(x-0)$

$\boxed{y = -0.731x + 1}$

**10.** [Maximum mark: 10]

A function $f$ is defined by $f(x) = (x+1)(x-1)(x-5)$, $x \in \mathbb{R}$.

(a) Find the values of $x$ for which $f(x) < |f(x)|$. [3]

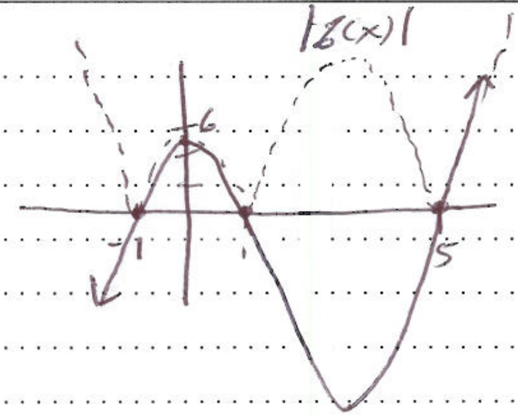
A function $g$ is defined by $g(x) = x^2 + x - 6$, $x \in \mathbb{R}$.

(b) Find the values of $x$ for which $g(x) < \dfrac{1}{g(x)}$. [7]

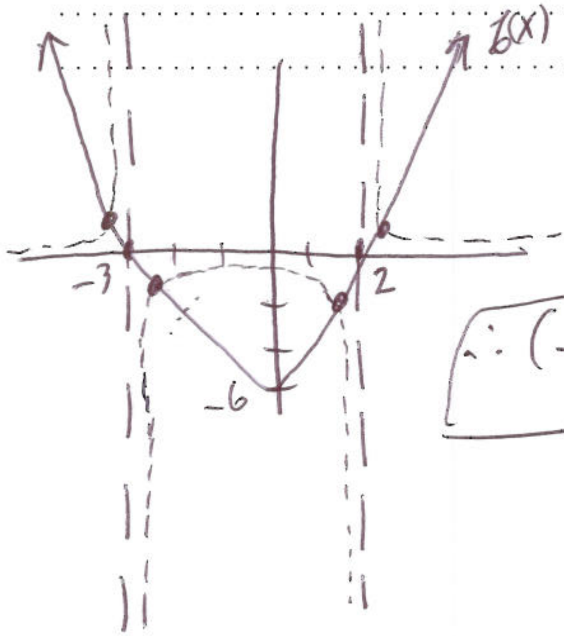
Create a fairly accurate sketch using GDC.

a) $y_1 = (x+1)(x-1)(x-5)$
$y_2 = |y_1|$ dashed lines

$(-\infty, -1) \cup (1, 5)$




b) $y_1 = x^2 + x - 6$ Solid
$y_2 = \dfrac{1}{(x^2+x-6)}$ dashed line but not including vertical asymptote @ $x = -3, 2$.




Find all 4 intersection points using GDC.
$x = -3.19, -2.79, 1.79, 2.19$

$\boxed{\therefore (-3.19, -3) \cup (-2.79, 1.79) \cup (2, 2.19)}$

Do **not** write solutions on this page.

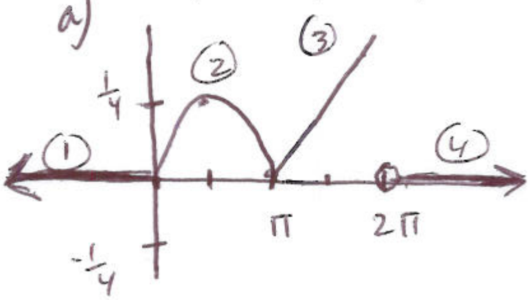
## Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

**11.** [Maximum mark: 20]

The probability density function of a continuous random variable $X$ is given by

a)




$$f(x) = \begin{cases} 0\ ,\ x < 0 & (1) \\ \dfrac{\sin x}{4}\ ,\ 0 \le x \le \pi & (2) \\ a(x-\pi),\ \pi < x \le 2\pi & (3) \\ 0\ ,\ 2\pi < x & (4) \end{cases}$$

(a) Sketch the graph of $y = f(x)$. [2]

(b) Find $\mathrm{P}(X \le \pi)$. $\int_0^{\pi} \frac{1}{4}\sin x\,dx = -\frac{1}{4}\cos x \Big|_0^{\pi} = \boxed{\frac{1}{2}}$ [2]

(c) Show that $a = \dfrac{1}{\pi^2}$. $\int_{-\infty}^{\infty} f(x)dx = 1$, so $\int_{-\infty}^{\pi} f(x)dx = \frac{1}{2}$ [3]

$\pi =$ (d) Write down the median of $X$. Thus $\int_{\pi}^{\infty} f(x)dx = \frac{1}{2}$ [1]

(e) Calculate the mean of $X$. $\int_{\pi}^{2\pi} f(x)dx = \frac{1}{2}$ use area of $\Delta$. [3]

(f) Calculate the variance of $X$. $A_{\Delta} = \frac{1}{2}bh$. Find $h$, when $x = 2\pi$ [3]

$a(x-\pi)$

$a(2\pi - \pi)$

(g) Find $\mathrm{P}\left(\dfrac{\pi}{2} \le X \le \dfrac{3\pi}{2}\right)$. $A_{\Delta} = \frac{1}{2}(\pi)(a\pi)$ $h = a\pi$ [2]

(h) Given that $\dfrac{\pi}{2} \le X \le \dfrac{3\pi}{2}$ find the probability that $\pi \le X \le 2\pi$. [4]

$\frac{1}{2}a\pi^2 = \frac{1}{2}$

$\therefore \boxed{a = \frac{1}{\pi^2}}$

e) $E(X) = \int_0^{\pi} x\,\frac{\sin x}{4}\,dx + \int_{\pi}^{2\pi} x\,\frac{(x-\pi)}{\pi^2}\,dx$

GDC: $E(X) = 3.40399\ldots$

$\boxed{E(X) = 3.40}$

11 f. $Var(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$Var(x) = \int_0^{\pi} x^2 \frac{\sin x}{4}\,dx + \int_{\pi}^{2\pi} x^2 \frac{(x-\pi)}{\pi^2}\,dx - (3.40399\ldots)^2$

GDC: $Var(x) = 3.866277\ldots = \boxed{3.87}$

g) Find $P(\frac{\pi}{2} \le X \le \frac{3\pi}{2})$

Method 1: Use Calculus

$\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{4}\,dx + \int_{\pi}^{\frac{3\pi}{2}} \frac{(x-\pi)}{\pi^2}\,dx = \boxed{0.375}$

Method 2: Use Geometry

$\frac{\sin x}{4}$ is symmetrical and area under curve from 0 to $\pi$ is $\frac{1}{2}$ $\therefore$ from $\frac{\pi}{2}$ to $\pi = \frac{1}{4}$

area under curve from $\pi$ to $\frac{3\pi}{2}$ is a triangle w/ $A = \frac{1}{2}bh$. Find $h$ when $X = \frac{3\pi}{2}$.

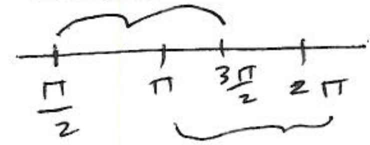
$f(\frac{3\pi}{2}) = \frac{1}{\pi^2}(\frac{3\pi}{2} - \pi) = \frac{1}{2\pi}$

$\therefore A = \frac{1}{2}(\frac{\pi}{2})(\frac{1}{2\pi}) = \frac{1}{8}$

Total area from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$ $= \frac{1}{4} + \frac{1}{8} = \boxed{\frac{3}{8} = 0.375}$

11 h. $P\left(\pi < x < 2\pi \;\middle|\; \frac{\pi}{2} < x < \frac{3\pi}{2}\right)$

$$= \frac{P\left(\pi < x < 2\pi \cap \frac{\pi}{2} < x < \frac{3\pi}{2}\right)}{P\left(\frac{\pi}{2} < x < \frac{3\pi}{2}\right)}$$ → Consider:




$$= \frac{P\left(\pi < x < \frac{3\pi}{2}\right)}{P\left(\frac{\pi}{2} < x < \frac{3\pi}{2}\right)} = \frac{\frac{1}{8}}{\frac{3}{8}} = \boxed{\frac{1}{3}}$$

Do **not** write solutions on this page.

**12.** [Maximum mark: 19]

(a) (i) Use the binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$.

(ii) Hence use De Moivre's theorem to prove

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta.$$

(iii) State a similar expression for $\cos 5\theta$ in terms of $\cos\theta$ and $\sin\theta$. [6]

Let $z = r(\cos\alpha + \mathrm{i}\sin\alpha)$, where $\alpha$ is measured in degrees, be the solution of $z^5 - 1 = 0$ which has the smallest positive argument.

(b) Find the value of $r$ and the value of $\alpha$. [4]

(c) Using (a) (ii) and your answer from (b) show that $16\sin^4\alpha - 20\sin^2\alpha + 5 = 0$. [4]

(d) Hence express $\sin 72^\circ$ in the form $\dfrac{\sqrt{a+b\sqrt{c}}}{d}$ where $a, b, c, d \in \mathbb{Z}$. [5]

a i) $\binom{5}{0}(\cos\theta)^5(i\sin\theta)^0 = \cos^5\theta$

$\binom{5}{1}(\cos\theta)^4(i\sin\theta)^1 = 5\cos^4\theta\, i\sin\theta$

$\binom{5}{2}(\cos\theta)^3(i\sin\theta)^2 = -10\cos^3\theta\sin^2\theta$

$\binom{5}{3}(\cos\theta)^2(i\sin\theta)^3 = -10\cos^2\theta\, i\sin^3\theta$

$\binom{5}{4}(\cos\theta)^1(i\sin\theta)^4 = 5\cos\theta\sin^4\theta$

$\binom{5}{5}(\cos\theta)^0(i\sin\theta)^5 = i\sin^5\theta$

$(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$

12 a ii $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$ DeMov.

$i\sin 5\theta$ belong to imaginary part. Equate imaginary part from (i) w/ each other.

$$\boxed{\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}$$

iii) $\cos 5\theta$ belong to the real part. Equate real part from (i) w/ each other.

$$\boxed{\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$$

12b $\quad z^5 - 1 = 0 \quad , \quad z = r \operatorname{cis} \alpha \quad$ Given.

$(r \operatorname{cis}\alpha)^5 - 1 = 0$

$r^5 \operatorname{cis} 5\alpha = 1$

$r^5 \operatorname{cis} 5\alpha = 1 \operatorname{cis} 0 \longrightarrow$ Note: 


$\hookrightarrow \cos 0 + i \sin 0$
$1 + i0 = 1$

$\therefore r^5 = 1 \quad , \quad \operatorname{cis} 5\alpha = \operatorname{cis} 0$

$\boxed{r = 1}$

$5\alpha = 0, \pm 360K, \quad K \in \mathbb{Z}$

$\alpha = \pm \frac{360}{5} K = \pm 72K$

$\therefore \boxed{\alpha = 72K}$ since smallest $K = 1$

$\therefore \alpha = 72^\circ$

c). $\sin 5\theta \Rightarrow \sin 5\alpha \Rightarrow \sin(5 \times 72)$

$\sin 5\theta$ is the imaginary part

$r^5 \operatorname{cis} 5\alpha = 1 \rightarrow r^5 \operatorname{cis} 5\alpha = 1 + 0i$ $\hookrightarrow$ 0 is the imaginary part of 1.

Equating imaginary parts:

$\sin(5 \times 72) = 0$

$0 = 5\cos^4\alpha \sin\alpha - 10\cos^2\alpha \sin^3\alpha + \sin^5\alpha \qquad$ from (a ii)

$0 = \sin\alpha\,(5\cos^4\alpha - 10\cos^2\alpha \sin^2\alpha + \sin^4\alpha)$

$\sin\alpha = 0 \quad$ or $\quad 5\cos^4\alpha - 10\cos^2\alpha \sin^2\alpha + \sin^4\alpha = 0$

$\sin\alpha \neq 0$

12 c continue

Convert $\cos^2\alpha$ to $\Rightarrow \cos^2\alpha = 1 - \sin^2\alpha$

$$5(1-\sin^2\alpha)^2 - 10(1-\sin^2\alpha)\sin^2\alpha + \sin^4\alpha = 0$$

$$5(1-2\sin^2\alpha+\sin^4\alpha) - 10\sin^2\alpha + 10\sin^4\alpha + \sin^4\alpha = 0$$

$$5 - 10\sin^2\alpha + 5\sin^4\alpha - 10\sin^2\alpha + 10\sin^4\alpha + \sin^4\alpha = 0$$

$$\boxed{16\sin^4\alpha - 20\sin^2\alpha + 5 = 0}$$

d) Factor part c. Let $u = \sin^2\alpha$

$$\therefore 16u^2 - 20u + 5 = 0$$

$$u = \frac{20 \pm \sqrt{20^2 - 4(16)(5)}}{32}$$

$$u = \frac{20 \pm \sqrt{80}}{32} = \frac{20 \pm 4\sqrt{5}}{32}$$

$$\sin^2\alpha = \frac{20 \pm 4\sqrt{5}}{32}$$

$$\sin\alpha = \pm\sqrt{\frac{20 \pm 4\sqrt{5}}{32}}$$

$$\sin\alpha = \pm\sqrt{\frac{20 \pm 4\sqrt{5}}{2\times 16}}$$

$$\sin\alpha = \frac{\pm\sqrt{10 \pm 2\sqrt{5}}}{\sqrt{16}}$$

$$\sin\alpha = \frac{\pm\sqrt{10 \pm 2\sqrt{5}}}{4}$$

sin 72 is in QI. and it is positive. We choose only the positive signs.

$$\boxed{\sin\alpha = \frac{\sqrt{10 + 2\sqrt{5}}}{4}}$$

Do **not** write solutions on this page.

**13.** [Maximum mark: 21]

Richard, a marine soldier, steps out of a stationary helicopter, $1000\,\mathrm{m}$ above the ground, at time $t = 0$. Let his height, in metres, above the ground be given by $s(t)$. For the first $10$ seconds his velocity, $v(t)\,\mathrm{ms}^{-1}$, is given by $v(t) = -10t$.

(a) (i) Find his acceleration $a(t)$ for $t < 10$. $a(t) = \frac{dv}{dt} = -10$ m/sec$^2$

(ii) Calculate $v(10) = -10(10) = -100$ m/sec.

(iii) Show that $s(10) = 500$. [6]

At $t = 10$ his parachute opens and his acceleration $a(t)$ is subsequently given by $a(t) = -10 - 5v$, $t \geq 10$. since $a(t) = \frac{dv}{dt}$

(b) Given that $\frac{\mathrm{d}t}{\mathrm{d}v} = \frac{1}{\frac{\mathrm{d}v}{\mathrm{d}t}}$, write down $\frac{\mathrm{d}t}{\mathrm{d}v}$ in terms of $v$. $\frac{dt}{dv} = \frac{1}{(-10-5v)}$ [1]

You are told that Richard's acceleration, $a(t) = -10 - 5v$, is always positive, for $t \geq 10$.

(c) Hence show that $t = 10 + \frac{1}{5}\ln\left(\frac{98}{-2-v}\right)$. [5]

(d) Hence find an expression for the velocity, $v$, for $t \geq 10$. [2]

(e) Find an expression for his height, $s$, above the ground for $t \geq 10$. [5]

(f) Find the value of $t$ when Richard lands on the ground. [2]

a iii $s(t) = \int v(t)\,dt = \int -10t\,dt = -5t^2 + C$

$s(t) = 1000$ Given. when $t = 0$

$\therefore 1000 = -5(0)^2 + c$

$1000 = c$

$\therefore \boxed{s(t) = -5t^2 + 1000}$

$s(10) = -5(10)^2 + 1000$

$\boxed{s(10) = 500}$

13c $\dfrac{dt}{dv} = \dfrac{1}{(-10-5v)}$ by separation.

$dt = \dfrac{1}{-10-5v} dt$ integrate both sides

$\int dt = \int \dfrac{1}{-10-5v} dt$

$t = \dfrac{\ln(-10-5v)}{-5} + c$ → Given: $t=10$, $v=-100$

$10 = \dfrac{\ln(-10-5(-100))}{-5} + c$

$10 = \dfrac{\ln 490}{-5} + c$

$\boxed{c = 10 - \frac{1}{5}\ln(490)}$

$\therefore \boxed{t = 10 - \frac{1}{5}\ln(490) - \frac{1}{5}\ln(-10-5v)}$

$t = 10 - \frac{1}{5}\left[\ln\left(\frac{490}{-10-5v}\right)\right]$

$t = 10 - \frac{1}{5}\ln\left(\frac{490}{5(-2-v)}\right)$

$\boxed{t = 10 - \frac{1}{5}\ln\left(\frac{98}{2-v}\right)}$

13 d) Solve for v in part c.

$t-10=\frac{1}{5}\ln\left(\frac{98}{-2-v}\right)$

$e^{[5(t-10)]}=e^{[\ln\frac{98}{-2-v}]}$

$\frac{98}{-2-v}=e^{5(t-10)}$

$-2-v=\frac{98}{e^{5(t-10)}}$

$\boxed{v=-\frac{98}{e^{5(t-10)}}-2}$

OR

$\boxed{v=-98e^{-5(t-10)}-2}$

e) $s(t)=\int v(t)\,dt$

$s(t)=\int\left(-98e^{-5(t-10)}-2\right)dt$

$s=\frac{-98e^{-5(t-10)}}{-5}-2t+c$

Given $s=500$, $t=10$

$500=\frac{-98e^{-5(10-10)}}{-5}-2(10)+c$

$500=\frac{98}{5}-20+c$

$\boxed{c=500.4}$

$\therefore$ $\boxed{s(t)=-2t+\frac{98}{5}e^{-5(t-10)}+500.4}$

f) Reach the ground has height, $s(t)=0$

$0=-2t+\frac{98}{5}e^{-5(t-10)}+500.4$

From GDC

$t=250.2$ when $s=0$

$\therefore$ $\boxed{t=250\text{ seconds.}}$